

Ultrarelativistic Bose-Einstein Gas on Lorentz Symmetry Violation

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Abstract

In this paper we study the effects of Lorentz Symmetry Breaking on thermodynamics properties of ideal gases. Inspired in the dispersion relation came from the Carroll-Field-Jackiw model for Electrodynamics with Lorentz and CPT violation term, we compute the thermodynamics quantities for a Boltzmann, Fermi-Dirac and Bose-Einstein distributions. Two regimes are analyzed: the non-relativistic and the relativistic one. In the first case we show that the topological mass induced by the Chern-Simons term behaves as a chemical potential. For the Bose-Einstein gases it could be found a condensation in both regimes, being the appearance of a condensate in the relativistic regime, the main contribution of this work.

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INTRODUCTION

Since the advent of the String Theories as a candidate to the Unified Theory, Lorentz- and CPT- Violations are expected at Planck Scale [1], and a background anisotropy on the space-time must correct the physics at a low energy scale. For instance, at a Standard Model scale, the Extended Standard Model SME [2] was proposed as a possible extension of the minimal Standard Model of the fundamental interactions. Even though the expectation of the Lorentz- and CPT violation, these effects are very small, and the SME has also been used as a framework to get stringent bounds on the Lorentz-symmetry violating (LV) coefficients [3], [4]. In this framework, there are a large number of results in the literature that investigate these effects in different situations, like system involving photons[5], [6], radiative corrections [7], fermions [8], neutrinos [9], topological defects [10], topological phases [11], cosmic rays [12], supersymmetry [13], particle decays [14], and other relevant aspects [15], [16].

This violation can be implemented in the fermion sector, for example, by $v_\mu \bar{\psi} \gamma^\mu \psi$, $b_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi$, leading to a modified Dirac theory [17]. It also has consequences in a very low energy regime, like in atomic physics, condensed matter and so on, taking into account the non relativistic limit, in order to obtain experimental bounds on the Lorentz symmetry braking (LSB) parameters and other effects, as the generation of an anomalous magnetic moment by a non-minimal coupling covariant derivative [18], [19].

In the gauge sector, it has been introduced by the pioneering work by Carrol, Field and Jackiw, modifying the Electromagnetic Maxwell Lagrangian by means of a Chern-Simons type extra term ($\propto \epsilon^{\mu\alpha\beta\gamma} v_\mu F_{\alpha\beta} A_\gamma$). This modify the dispersion relation of the photon introducing a topological mass and preserving the gauge invariance. It has been pointed out by Perez-Victoria [7] that this term is a one loop quantum correction of the fermion coupling to the background.

Even though LSB is a theoretical implication, the search for a deviation on the photon dispersion relation has important consequences on cosmic physics, like Gamma Ray Bursts, near stars with strong magnetic field and vacuum birefringence. This can justify the study of the statistical behavior of the photon gas under a non-linear dispersion relation. As this violation is expected to be observed only at ultra high energy, this address to the question of its statistical behavior in a ultra-relativistic photon gas, not only to get more stringent bounds on the LSB parameters but also an interesting phenomenon that could be related

by this breaking.

Another interesting point is related to the light bosons [23], which are described by a scalar field, when minimally coupled to gravity it could be a candidate for Dark Matter. Its mass is constrained to the order $10^{-22}eV$. In ref. [23], the authors argue that the neutrino radiation behavior is linked to a ultra-relativistic transition. This scalar field falls in the classification of Hot Dark Matter (HDM), in the sense that it behaves as radiation at its decoupling epoch. This is related to a Bose-Einstein condensation in ultra-relativistic gas. These facts motivate the study of a Bose-Einstein gas with a non-linear dispersion relation at the ultra-relativistic level.

In this paper, we study the statistical mechanics of an ideal gas embedded in a LSB background, starting from a massive Chern-Simons dispersion relation. Two regimes are computed. In the non relativistic, when the background is larger than the momentum of the particle, we will show that in this limit our results are the same obtained when started by the SME Lagrangian where a non-relativistic Hamiltonian is computed for the free the Bose-Einstein, Fermi Dirac and Boltzmann gas [31]. Our contribution in this paper is the study of the relativistic gas. We shall see, for the Bose gas, that LSB background can induce a phase transition for the high energy gas.

I. BOLTZMANN'S GAS

As a first step, we analyze the Statistical Mechanics for an ideal Boltzmann gas, by computing the relativistic and the non-relativistic limit. The gas is described by the dispersion relation raised from Carroll-Field-Jackiw model, which is given by, $p^4 + v^2 p^2 - (v.p)^2 = 0$, where ν is a four-vector background field. This background vector violates CPT and Lorentz symmetries in the particle frame. We take the space-like vector that obey the causal conditions described in [32]. In this regime, E is given by

$$E = \sqrt{\frac{2\vec{p}^2 + \vec{v}^2}{2}} \pm \frac{1}{2}\sqrt{\vec{v}^4 + 4(\vec{v}.\vec{p})^2}. \quad (1)$$

In statistical mechanics the partition function for a system of N particles is given by

$$\Xi(T, V, \mu) = \sum_{N=0}^{\infty} z^N Z(T, V, N) \quad (2)$$

where $Z(T, V, N) = \frac{1}{N!} Z^N(T, V, 1)$ and $Z(T, V, 1) = \frac{1}{h^3} \int e^{-\beta E(p, q)} dp dq$ with $\beta = \frac{1}{kT}$ and $z = e^{\beta\mu}$. In order to split the two regimes that we are interested in, we analyze the limit cases, when $|\vec{v}| \gg |\vec{p}|$, that is the non-relativistic regime of the (1)

$$E_{\pm} = |\vec{v}| + \frac{1}{2} \frac{p^2}{|\vec{v}|} (1 \pm \cos^2 \theta) \quad (3)$$

and the relativistic case, $|\vec{v}| \ll |\vec{p}|$, the first order on v of (1) assumes

$$E = |\vec{p}| + \frac{1}{2} |\cos \theta| v, \quad (4)$$

A. The non-relativistic Limit: $|\vec{v}| \gg |\vec{p}|$

In order to study the statistical mechanics of the classical gas in the non-relativistic limit, we put the dispersion relation (3) in the partition function (2). Here, we will not consider the case with negative energy, by means that this situation violates the causality conditions, [32]. Then,

$$Z(T, V, 1) = \frac{2\pi V e^{-\beta v}}{h^3} \int_0^\pi \int_0^\infty \sin \theta e^{-\beta [\frac{1}{2} \frac{p^2}{v} (1 + \cos^2 \theta)]} p^2 dp d\theta, \quad (5)$$

after a straightforward integration, we obtain the Z -function $Z(T, V, 1) = \frac{1}{4} \left(\frac{2v}{2\pi\beta} \right)^{3/2} V e^{-\beta v}$.

Thus, the partition function is

$$\Xi(T, V, \mu) = \sum_{N=0}^{\infty} \frac{1}{N! 4^N} \left(\frac{2v}{2\pi\beta} \right)^{\frac{3}{2}N} V^N e^{\beta(\mu-v)N}. \quad (6)$$

To compute the number of particles $N = -\frac{\partial \phi}{\partial \mu}$, pressure $P = -\frac{\partial \phi}{\partial V}$ and the entropy $S = -\frac{\partial \phi}{\partial T}$, we calculate the grand-potential function

$$\phi = -kT \ln \Xi = -kT e^{\beta(\mu-v)} \frac{1}{4} \left(\frac{2v}{2\pi\beta} \right)^{3/2} V, \quad (7)$$

that results, respectively

$$\begin{aligned} N &= e^{\beta(\mu-v)} \frac{1}{4} \left(\frac{2v}{2\pi\beta} \right)^{3/2} V, \\ P &= \frac{kTN}{V}, \\ S &= kN \left[\frac{5}{2} - (\mu - v) \beta \right]. \end{aligned} \quad (8)$$

The energy can be computed by the relation, $U = TS - PV + \mu N$ which results in $U = \frac{3}{2}NkT + vN$, and the state equation $U = (\frac{3}{2} + \beta v) PV$. The chemical potential can be calculated explicitly in this case as

$$\mu = -kT \left[\ln \left(\frac{N_Q}{N_C} \right) - \left(\frac{v}{kT} + \ln 4 - 1 \right) \right], \quad (9)$$

where we call the quantities $N_Q = \left(\frac{2v}{2\pi\beta} \right)^{\frac{3}{2}}$ as the quantum concentration and $N_C = \frac{N}{V}$ the standard classical concentration, or density of particles. Note that the chemical potential is affected by the background field v as was reported by Colladay *et al* [31]. This change is due to the behavior of the field as an effective mass for the gas.

In the thermodynamic limit, and when $kT \gg v$, we recover the non-relativistic limit result to the photon gas without Lorentz-violation and the specific heat

$$C_V = \frac{\partial U}{\partial T} = \frac{3}{2}Nk. \quad (10)$$

Thus, our approach recover the standard results when the background vector v is neglected for the non relativistic gas. In this situation, it is not interesting to investigate bounds on the LSB parameters because we are describing here a classical non-relativistic photon gas.

B. The Relativistic Limit $|\vec{v}| \ll |\vec{p}|$,

By the same procedure used in the non-relativistic limit, we find for the Z -function $Z(T, V, 1) = \frac{8V}{(2\pi)^2 \beta^4 v} e^{-\frac{\beta v}{2}} (e^{\frac{\beta v}{2}} - 1)$, that the partition function and the grand potential become

$$\begin{aligned} \Xi(T, V, \mu) &= \exp \left[e^{\beta\mu} \frac{8V}{(2\pi)^2 \beta^4 v} e^{-\frac{\beta v}{2}} (e^{\frac{\beta v}{2}} - 1) \right] \\ \phi &= -kT \left[e^{\beta\mu} \frac{8V}{(2\pi)^2 \beta^4 v} e^{-\frac{\beta v}{2}} (e^{\frac{\beta v}{2}} - 1) \right] \end{aligned} \quad (11)$$

Then the particle number is

$$N = e^{\beta\mu} \frac{8V}{(2\pi)^2 \beta^4 v} e^{-\frac{\beta v}{2}} (e^{\frac{\beta v}{2}} - 1).$$

The entropy, energy and the specific heat C_V with the background are respectively

$$S = kN \left[5 - \mu\beta - \frac{v}{2}\beta - \frac{v}{2}\beta \left(\frac{e^{\frac{\beta v}{2}}}{e^{\frac{\beta v}{2}} - 1} \right) \right] \quad (12)$$

$$U = \left[4 - \left(\frac{\frac{\beta v}{2}}{e^{\frac{\beta v}{2}} - 1} \right) \right] PV \quad (13)$$

$$C_V = 4kN - \frac{e^{\frac{v}{2kT}} N v^2}{4(e^{\frac{v}{2kT}} - 1)kT^2}. \quad (14)$$

and we have the same state equation $PV = NkT$.

The chemical potential is now given by

$$\mu = kT \left[\ln \left(\frac{\pi^2 \beta^4 v}{2N_C} \right) - \left(\frac{v}{kT} + \ln \left(1 - e^{-\frac{\beta v}{2}} \right) - 1 \right) \right]. \quad (15)$$

We can observe how the background parameter v modify the statistics. In the limit which the background does not exist, we recover the standard result $U = 3NkT$ and $C_v = 3Nk$. The behavior of the Boltzmann gas in the presence of background field is illustrated in the figures that follow. In Fig.1 we can observe the behavior of the internal energy with the temperature and a fixed parameter v . For large temperatures all the curves tend asymptotically to the value $U = 3NkT$ and collapse to this value when the parameter v goes to zero. The same behavior can be seen in Fig.2, where U is a function of v . Note here that the field v clearly introduces an effective mass in the system so that the internal energy increases. The behavior of the specific heat is shown in Fig.3 and again we found the change introduced by the background field. When v goes to zero at $T = 0$ the value of $C_v = 3Nk$, the same expected by Boltzmann's statistics.

II. FERMI-DIRAC STATISTICS

We now begin to study the effects of the LSB background on the quantum gases. To this end, we should compute the Fermi-Dirac and Bose-Einstein distribution and in this paper we are interested in both limits, non-relativistic and relativistic regime. In the Fermi-Dirac statistics the grand potential is given by

$$\phi = -kT \frac{2\pi V \beta}{h^3} \frac{1}{3} \int_0^\pi \int_0^\infty \sin \theta p^3 \frac{ze^{-\beta\epsilon}}{1 + ze^{-\beta\epsilon}} \left(\frac{d\epsilon}{dp} \right) dp d\theta. \quad (16)$$

The non relativistic regime, i.e. $|\vec{v}| \gg |\vec{p}|$, $\epsilon = |\vec{v}| + \frac{1}{2} \frac{p^2}{v} (1 + \cos^2 \theta)$ and $\frac{d\epsilon}{dp} = \frac{p}{v} (1 + \cos^2 \theta)$, which yields the grand partition function

$$\phi = -kT \frac{V}{\lambda^3} \frac{\sqrt{2}}{2} f_{5/2}(ze^{-\beta v}) \quad (17)$$

We should note that the thermal wave length of the Fermi gas λ , depends on the LSB parameter

$$\lambda = \left(\frac{h^2}{2\pi v kT} \right)^{1/2} \quad (18)$$

and $f_n(\chi)$ is the complete Fermi-Dirac integral defined by

$$f_n(\chi) = \frac{1}{\Gamma(n)} \int_0^\infty \frac{\xi^{n-1}}{\chi^{-1}e^\xi + 1} d\xi. \quad (19)$$

Some important thermodynamic quantities as particle number, pressure and density of energy is defined below:

$$\begin{aligned} N &= \frac{V}{\lambda^3} \frac{\sqrt{2}}{2} f_{3/2}(ze^{-\beta v}) \\ P &= \frac{1}{\lambda^3} (kT) \frac{\sqrt{2}}{2} f_{5/2}(ze^{-\beta v}) \\ U &= -\frac{1}{V} \frac{\partial \phi}{\partial \beta} = \frac{1}{\lambda^3} \frac{\sqrt{2}}{2} \left[v f_{3/2}(ze^{-\beta v}) + \frac{3}{2} (kT) f_{5/2}(ze^{-\beta v}) \right] V \end{aligned} \quad (20)$$

The the equation of state for a Fermi gas is

$$PV = NkT \frac{f_{5/2}(ze^{-\beta v})}{f_{3/2}(ze^{-\beta v})}.$$

For high temperatures the Fermi's gas behaves like a Boltzmann's gas, then the complete Fermi-Dirac function is $f_n(\chi) \approx \chi$, or $PV = NkT$. These results are the same obtained by Colladay [31]

The relativistic regime, i.e. $|\vec{v}| \ll |\vec{p}|$ and the energy $\epsilon = |\vec{p}| + \frac{1}{2} |\cos \theta| v$, $\frac{d\epsilon}{dp} = 1$. The grand partition function can be written as

$$\phi = -kT \frac{8\pi V}{h^3} \frac{1}{\beta^3} I_5(z, \chi) \quad (21)$$

where the functions $I_n(z\chi)$ are

$$I_n(z, \chi) = \frac{1}{\ln \chi} (f_n(z) - f_n(z\chi)) \quad (22)$$

In the limit when $v \rightarrow 0$ the grand potential becomes

$$\phi = -kT \frac{8\pi V}{h^3} \frac{1}{\beta^3} f_4(z) \quad (23)$$

The pressure may be written like

$$P = \frac{8\pi}{h^3} (kT)^4 I_5(z, \chi) \quad (24)$$

For $v \rightarrow 0$ and $kT \gg \mu$, we have

$$P = \frac{8\pi}{h^3} (kT)^4 \frac{\pi^4}{90} \frac{7}{8}$$

We see that the pressure for Fermi-Dirac gas differs by the factor $\frac{7}{8}$ when compared to Bose gas in same condition $v \rightarrow 0$ and $z \rightarrow 1$ as usual.

The internal energy U is

$$U = \frac{8\pi V}{h^3} (kT)^4 (4I_5(z, \chi) - f_4(z\chi))$$

when $v \rightarrow 0$ and $z \rightarrow 1$

$$U = \frac{8\pi V}{h^3} (kT)^4 \frac{7\pi^4}{240} \quad (25)$$

III. BOSE-EINSTEIN STATISTICS

Colladay *et. al* have been studied the effects of the LSB background in a general non-relativistic statistics. The general effects of the background in this kind of situation is to redefine the thermodynamics quantities, unlike to obtain a new effect, since the background is expected to be small in low energy scale. In this framework, these systems can be used to get experimental bounds on the LSB parameters. In a Bose gas Colladay *et. al* have been analyzed in two very different situation . In a first publication, it has been shown how the

background can modify the standard thermodynamics results as the critical temperature for the homogeneous non relativistic Bose-Einstein Condensates (BEC) phase transition, starting from the non-relativistic Hamiltonian with LSB non-relativistic contribution. In a second publication it has been proposed to use the BEC trapped as probe to LSB parameters.

Here, we start from the Carroll-Field Jackiw dispersion relation (1), and compute both limits. The non-relativistic regime by the same way that was computed to the Boltzmann Gas (3). We will show that our results are the same obtained by Colladay *et. al*, using other method. Our goal here, is the study of the ultra-relativistic free Bose gas under the same background. We will see that under this regime, a phase transition can be induced by the vector background.

The grand canonical partition function can be calculated evaluating the number of states in the one-particle phase space Σ , where in spherical coordinates is $\Sigma = \frac{2\pi V}{h^3} \int_0^\pi \sin \theta d\theta \int_0^\infty p^2 dp$, the one-particle density of states is given by

$$g(\epsilon) = \frac{d\Sigma}{d\epsilon} = \left[\frac{2\pi V}{h^3} \int_0^\pi \sin \theta \int_0^\infty p^2 dp \right] \frac{dp}{d\epsilon}. \quad (26)$$

The macro canonical partition function Ξ in Bose statistics is given by

$$\ln \Xi(T, V, z) = - \sum_k^\infty \ln(1 - ze^{-\beta\epsilon_k}). \quad (27)$$

By computing the thermodynamic limit, $\ln \Xi = - \int_0^\infty g(\epsilon) \ln(1 - ze^{-\beta\epsilon}) d\epsilon - \ln(1 - z)$, then grand potential is

$$\phi = -kT \frac{2\pi V}{h^3} \frac{\beta}{3} \int_0^\pi \int_0^\infty \sin \theta p^3 \frac{ze^{-\beta\epsilon}}{1 - ze^{-\beta\epsilon}} \left(\frac{d\epsilon}{dp} \right) dp d\theta - \ln(1 - z) \quad (28)$$

where $z = e^{\beta\mu}$ is the fugacity.

A. The non-relativistic Limit $|\vec{v}| \gg |\vec{p}|$:

The dispersion relation in the non-relativistic limit is given by (3). To study the effects of LSB in a non-relativistic Bose gas, we put this dispersion relation in (28). In this situation, the grand potential function can be written as

$$\phi = -kT \frac{2\pi V}{h^3} \frac{\beta}{3} \int_0^\pi \int_0^\infty \sin \theta p^3 \frac{ze^{-\beta \left[v + \frac{1}{2} \frac{p^2}{v} (1 + \cos^2 \theta) \right]}}{1 - ze^{-\beta \left[v + \frac{1}{2} \frac{p^2}{v} (1 + \cos^2 \theta) \right]}} \frac{p}{v} (1 + \cos^2 \theta) dp d\theta \quad (29)$$

after evaluating the integral, we obtain

$$\phi = -kT \frac{V}{\lambda^3} \frac{\sqrt{2}}{2} g_{5/2}(e^{\beta(\mu-v)}) \quad (30)$$

where $g_n(z)$, is defined by $g_n(z) = \frac{1}{\Gamma(n)} \int_0^\infty \frac{x^{n-1} dx}{z^{-1}e^x - 1}$, $0 \leq z \leq 1$ and $\lambda = \left(\frac{h^2}{2\pi v k T}\right)^{1/2} = \left(\frac{2\pi}{v k T}\right)^{1/2}$, is the thermal wavelength.

Computing the thermodynamical quantities, we obtain

$$\begin{aligned} P &= \frac{1}{\lambda^3} (kT) \frac{\sqrt{2}}{2} g_{5/2}(e^{\beta(\mu-v)}), \\ N &= \frac{V}{\lambda^3} \frac{\sqrt{2}}{2} g_{3/2}(e^{\beta(\mu-v)}) + N_0 \end{aligned} \quad (31)$$

The total number N is then $N = N_1 + N_0$ where N_0 is the number of particles in the ground state and $N_1 = \frac{V}{\lambda^3} \frac{\sqrt{2}}{2} g_{3/2}(e^{\beta(\mu-v)})$ is the number of particles in excited states.

The critical temperature T_c when the Bose-Einstein condensation (BEC) occurs is obtained when $g_{3/2}(x)$ has a maximum, or $\mu = v$.

$$T_c = \left(\frac{N}{V}\right)^{2/3} \frac{h^2}{2\pi v k} \frac{1}{(2\zeta(3/2)^2)^{1/3}}, \quad (32)$$

the number of particles below of the critical temperature, or $T < T_c$ is

$$N = N_1 = \frac{V}{\lambda^3} \frac{\sqrt{2}}{2} \zeta(3/2). \quad (33)$$

For $T > T_c$ we have

$$N = N_1 = \frac{V}{\lambda^3} \frac{\sqrt{2}}{2} g_{3/2}(e^{\beta(\mu-v)}), \quad (34)$$

the remaining particles are in ground state, so we can write

$$\frac{N}{N_0} = \frac{N - N_1}{N} = 1 - \left(\frac{T}{T_c}\right)^{3/2}. \quad (35)$$

The internal energy U for $T < T_c$ is

$$U = N_1(kT) \left[\frac{v}{kT} + \frac{3}{2} \frac{g_{5/2}(1)}{g_{3/2}(1)} \right], \quad (36)$$

Using the equation eq. (35) the specific heat is $C_V = \frac{15}{4} Nk \frac{\zeta(5/2)}{\zeta(3/2)} \left(\frac{T}{T_c} \right)^{3/2}$. Above of the critical temperature, $T > T_c$, the internal energy U assumes

$$U = N(kT) \left[\frac{3}{2} \frac{g_{5/2}(z)}{g_{3/2}(z)} \right], \quad (37)$$

where z is now set as $z = e^{\beta(\mu-v)}$. The specific heat reads

$$C_V = \frac{3}{2} Nk \left[\frac{5}{2} \frac{g_{5/2}(z)}{g_{3/2}(z)} - \frac{3}{2} \frac{g_{3/2}(z)}{g_{1/2}(z)} \right] \quad (38)$$

B. The Ultra-relativistic Bose Gas: $|\vec{p}| \gg |\vec{v}|$

In this section, the ultra-relativistic ideal Bose gas is studied, starting from the dispersion relation Carrol-Field-Jackiw (4) in the Bose-Einstein distribution 28. We will see that the background modifies the thermodynamics of the gas, introducing a phase transition that does not exist in the standard Bose gas. This effect is very interesting and this result could be important for astrophysical and cosmological applications [23, 35].

The grand potential is

$$\phi = -kT \frac{2\pi V}{h^3} \frac{\beta}{3} \int_0^\pi \int_0^\infty \sin \theta p^3 \frac{z e^{-\beta[p + \frac{1}{2} |\cos \theta|]}}{1 - z e^{-\beta[p + \frac{1}{2} |\cos \theta| v]}} dp d\theta \quad (39)$$

Performing the integration over θ and p we obtain

$$\phi = -kT \frac{8\pi V}{h^3} \frac{1}{\beta^3} \left[\frac{2}{\beta v} (g_5(e^{-\beta\mu}) - g_5(e^{-\beta(\mu - \frac{v}{2})}) \right]. \quad (40)$$

In the limit $v \rightarrow 0$ and $\mu = 0$ we have the standard photon gas, the function $g_5(e^{-\frac{\beta v}{2}}) \rightarrow \zeta(5)$, and the grand potential for the ultrarelativistic Bose-Einstein gas, in the limit where the LSB background does not exist, becomes the well-known result

$$\phi = -kT \frac{8\pi V}{h^3} \frac{1}{\beta^3} g_4(1), \quad (41)$$

with $g_4(1) = \frac{\pi^4}{90}$.

For further calculations we will write now the grand potential as

$$\phi = -kT \frac{8\pi V}{h^3} \frac{1}{\beta^3} F_5(z, v), \quad (42)$$

where the function $F_5(z, v)$ will be definite by

$$F_5(z, v) = \frac{2}{\beta v} (g_5(z) - g_5(ze^{-\beta \frac{v}{2}})) \quad (43)$$

and is easy to show that its derivative with respect to β is given by

$$\frac{\partial F_5(z, v)}{\partial \beta} = \frac{1}{\beta} (g_4(ze^{-\beta \frac{v}{2}}) - F_5(z, v)). \quad (44)$$

The thermodynamical quantities are given by

$$P = \frac{8\pi}{h^3} (kT)^4 F_5(z, v). \quad (45)$$

When $v \rightarrow 0$ we obtain the well-known result for ultrarelativistic Bose-Einstein gas $P = \frac{8\pi}{h^3} (kT)^4 \frac{\pi^4}{90}$.

The internal energy can be calculated by

$$U = \frac{8\pi V}{h^3} (kT)^4 [4F_5(z, v) - g_4(ze^{-\beta \frac{v}{2}})], \quad (46)$$

the state equation is then

$$\frac{U}{V} = \frac{4F_5(z, v) - g_4(ze^{-\beta \frac{v}{2}})}{F_5(z, v)} P. \quad (47)$$

When $v \rightarrow 0$ the energy density is

$$\frac{U}{V} = 3P \quad (48)$$

In the above equations we can observe that the pressure and the energy density depends only of the temperature and it vanishes as T^4 for $T \rightarrow 0$. In PV diagrams the isotherms are parallel lines to V-axis.

C. The Mean Particle Number and the Bose Temperature

A very useful quantity is the mean particle number, that in Bose-Einstein statistics is given by $N(T, V, z) = \sum_k \frac{1}{z^{-1}e^{\beta \epsilon_k} - 1}$. In the thermodynamic limit we have

$$N = \frac{2\pi V}{h^3} \int_0^\pi \int_0^\infty \sin \theta p^2 \frac{ze^{-\beta[p+\frac{1}{2}|\cos \theta|v]}}{1 - ze^{-\beta[p+\frac{1}{2}|\cos \theta|v]}} dp d\theta \quad (49)$$

or explicitly

$$N = \frac{8\pi V}{h^3} \frac{1}{\beta^3} \frac{2}{\beta v} (g_4(z) - g_4(ze^{-\beta \frac{v}{2}})) \quad (50)$$

The maximum mean number of particle is defined when $\mu = \frac{v}{2}$ or when we reach the critical temperature T_c . In that temperature we have

$$N = \frac{8\pi V}{h^3} \frac{1}{\beta^3} \frac{2}{\beta v} (g_4(e^{-\beta \frac{v}{2}}) - \zeta(4)). \quad (51)$$

Expanding the above equation for $\frac{v}{2kT} \ll 1$ and taking only terms in first order of v we have

$$N = \frac{8\pi V}{h^3} (kT)^3 \left(\zeta(3) + \frac{\pi^2}{12} \frac{v}{2kT} \right) \quad (52)$$

The critical temperature T_c , where the Bose Einstein Condensation occurs is the real cube root of the Eq.(52). The necessary condition for T_c real is

$$v \leq \frac{18(16\pi^2\zeta(3)^2)^{1/3} \hbar}{\pi^2} \left(\frac{N}{V} \right)^{1/3}, \quad (53)$$

where N/V is the density of the condensate. The maximum mass of v so that the gas is a condensate in MeV/c^2 is

$$v \leq 0.049465 \rho^{1/4}, \quad (54)$$

where ρ is now the density of the gas in kg/m^3 .

When $T = T_c$ the energy of the gas

$$U = NkT \left(\frac{4F_5(z, v) - g_4(ze^{-\beta \frac{v}{2}})}{F_4(z, v)} \right) \quad (55)$$

becomes

$$U = N_1 kT \left\{ \frac{4(g_5(e^{\beta \frac{v}{2}}) - \zeta(5)) - \frac{\beta v}{2} \zeta(4)}{g_4(e^{\beta \frac{v}{2}}) - \zeta(4)} \right\} \quad (56)$$

and the specific heat is given by

$$C_v = 4Nk \left(\frac{T}{T_c} \right)^3 \left\{ \frac{4(g_5(e^{\beta \frac{v}{2}}) - \zeta(5)) - \frac{\beta v}{2} \zeta(4)}{g_4(e^{\beta \frac{v}{2}}) - \zeta(4)} \right\} + (kT) \left(\frac{T}{T_c} \right)^3 \frac{\partial}{\partial T} \left\{ \frac{4(g_5(e^{\beta \frac{v}{2}}) - \zeta(5)) - \frac{\beta v}{2} \zeta(4)}{g_4(e^{\beta \frac{v}{2}}) - \zeta(4)} \right\} \quad (57)$$

The standard case (photon gas and absence of background field) is described in the limit $v \rightarrow 0$.

$$\frac{C_V}{Nk} = \frac{3\zeta(4)}{\zeta(3)} \approx 2.70118 \quad (58)$$

D. The BEC of an Ultrarelativistic Bose Gas with a Conserved Quantum Number

The behavior of the Bose-Einstein condensate in high temperatures when we have an ideal gas with a conserved quantum number, or generically referred as “charge” is quite different from that studied so far. Haber and Weldon [34] for the first time showed the effects on the critical temperature when antiparticles are introduced in the theory.

The net charge Q of the Bose gas is given by the expression

$$Q = V \sum_k \left[\frac{1}{e^{\beta(E_k - \mu)}} - \frac{1}{e^{\beta(E_k + \mu)}} \right] \quad (59)$$

In our case, using the dispersion relation given in Eq. (4) and taking the thermodynamic limit in Eq. (59), we find

$$\rho = \frac{N}{V} = \frac{2\pi}{h^3} \int_0^\pi \int_0^\infty \sin \theta p^2 \left(\frac{1}{e^{\beta[p + \frac{1}{2}|\cos \theta|v - \mu]} - 1} - \frac{1}{e^{\beta[p + \frac{1}{2}|\cos \theta|v + \mu]} - 1} \right) dp d\theta \quad (60)$$

After integrating over p and θ we obtain the explicit result in terms of poly-logarithm functions

$$\rho = \frac{8\pi}{h^3} (kT)^3 \frac{2}{\beta v} \left[g_4(e^{\beta\mu}) - g_4(e^{-\beta\mu}) - g_4(e^{\beta(\mu - v/2)}) + g_4(e^{-\beta(\mu + v/2)}) \right]. \quad (61)$$

In the regime of high temperatures when the pair creation is very favorable, $\frac{v}{2kT} \ll 1$. The BEC occurs when $\mu = \frac{v}{2}$, and net density of charge for the critical temperature T_c reads

$$T_c = \sqrt{\frac{h^3}{4\pi^3 k^2} \frac{3|\rho|}{v}} \quad (62)$$

This result is the same obtained for the authors [34] in their seminal work, two decades ago. The new critical temperature depends now directly of the parameter v .

IV. FINAL COMMENTS

In this paper we have analyzed how the Lorentz Symmetry breaking background modify the statistical behavior of a many particle system starting from the Carrol-Field-Jackiw dispersion relation. We have computed particles in classical and quantum regime, and special attention was devoted to Bose-Einstein statistics in relativistic and non-relativistic approach. It was pointed out that in the non-relativistic regime our approach gives raise the same results obtained by Colladay *et al*, where the thermodynamics quantities must be corrected by the presence of v^μ . These results open up the possibilities in bounds on the LSB parameters.

An interesting scenario is the ultra-relativistic thermodynamic approach, where v^μ induce a relativistic phase transition of a system with bosons. In the paper of ref [23], it is argued that the radiation behavior of hot dark matter is close related to a relativistic Bose-Einstein phase transition of a scalar charged field when coupled to gravity, even though this relation is not very well understood. [35]. The similarities in the behavior of SFDM BEC and the Ultra-Relativistic Bose gas under a LSB background could be pointed out as the LSB parameter that is responsible to the phase transition is also very small. In our case, the origin of this term is completely different as well as it is not scalar field, but vectorial. It should be pointed out that the mass of these particles are related to the medium, in the same sense of in condensed matter. A background renormalizes the photon mass, and it works like an effective mass.

It is interesting to note that the critical value of the energy density of condensate is related to the LSB parameter by $v \leq 0.049465 \rho^{1/4}$. This could be used for the different universe radiation eras to fix the LSB parameter taking account the red-shift of the quantities due to the big-bang expansion.

Another important result is about the Eq.(62), which relates the critical temperature with the LSB parameter. This could be used to calculate the density of energy and pressure of different species of particles.

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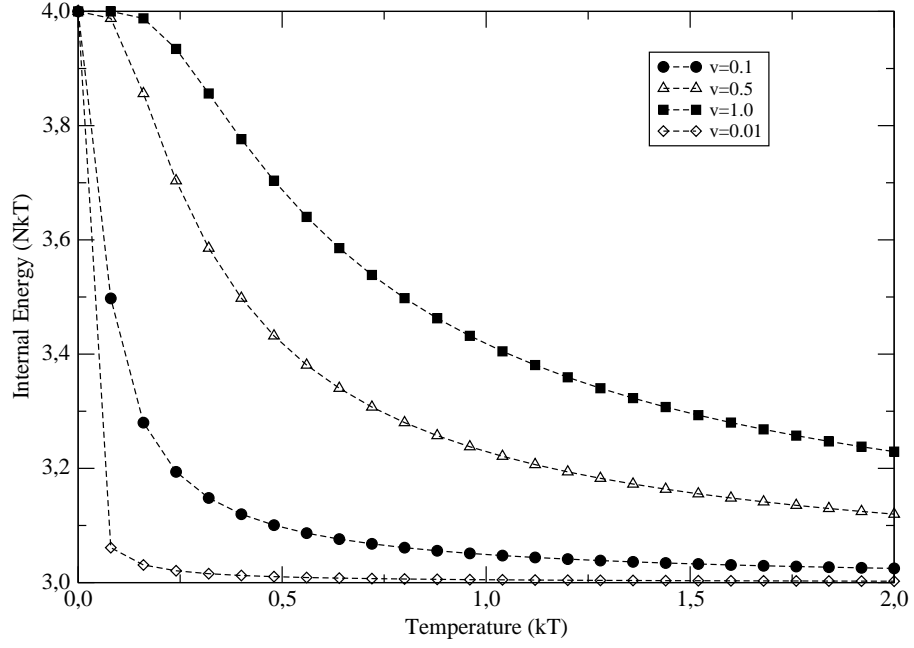


Figure 1: Internal Energy U as a function of kT . We used values $v = 1.0, 0.5, 0.1, 0.01$. When $v \rightarrow 0$ all curves will degenerate to $U = 3NkT$ for any temperature.

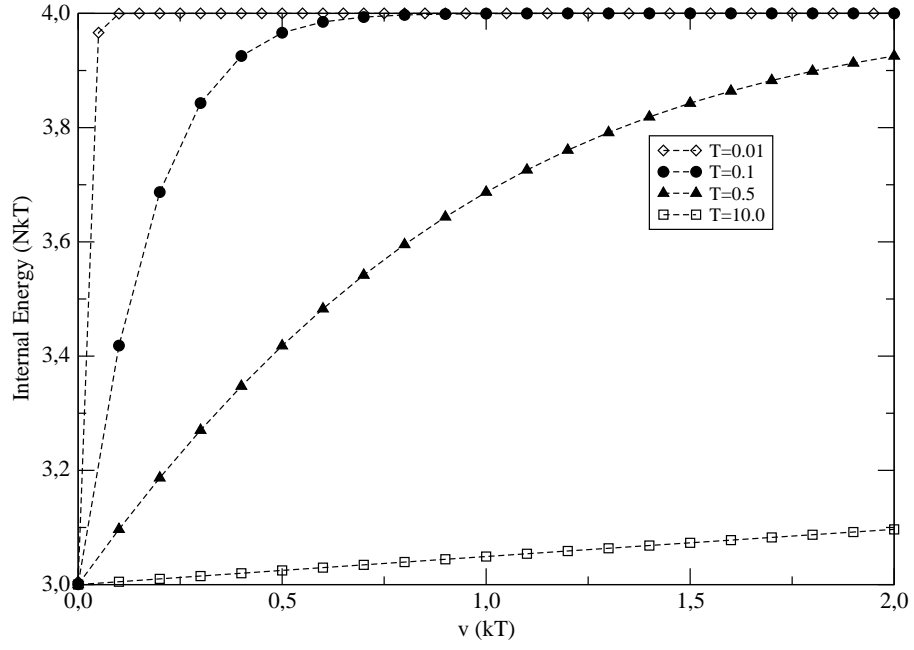


Figure 2: Internal Energy U as a function of v . We used values $T = 0.01, 0.1, 0.5, 10.0$. When $v \rightarrow 0$ all curves converge to $U = 3NkT$ for any temperature.

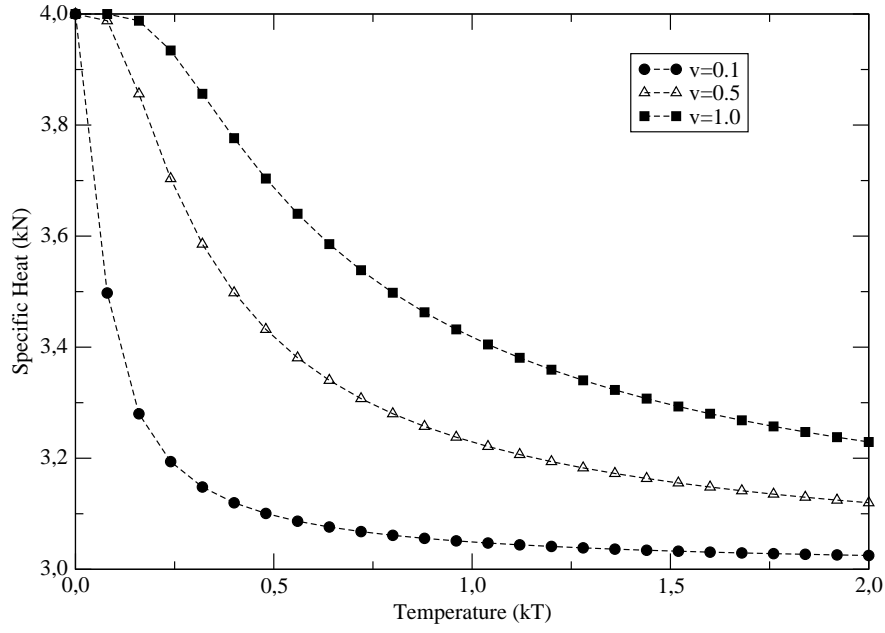


Figure 3: Specific Heat C_v as a function of T . We used values $v = 1, 0.5, 0.1$. When $v \rightarrow 0$ all curves converge to $C_v = 3kN$ for any temperature.